

Applying a water pinch analysis technique to reduce the water consumption at a paper mill

T.N. Iswalal, C.A Buckley, I. Kerr

Pollution research group, University of KwaZulu-Natal, South Africa

This paper presents the research findings from applying a water pinch analysis technique to systematically reduce the specific water consumption at a South African paper mill. It details the various pinch analysis techniques applicable to the paper mill investigated, the requirements with respect to the contaminants in the system, the data required, the application of the most suitable method to the system and the significant results obtained. The limitations of the method are highlighted as well as recommendations for the way forward once the water pinch analysis is complete.

Keywords: water reduction, water pinch analysis

1. Introduction

The efficient use of resources has become a major concern in most production industries due to increasing costs and stringent environmental regulations. Process integration is becoming an attractive solution to determine the most efficient reuse and recycle of resources within an operating system. Process integration techniques initially developed for energy integration have been modified to be applicable to water reuse-recycle systems with major developments being in the past decade.

Various techniques exist for both the fixed-rate and fixed-load type of problem; the paper making process has been identified as a fixed-rate type of problem and hence only these methods are highlighted in this paper.

Many methods are presented for targeting for water reuse and recycle. The first method for flowrate targeting for the fixed load problem was the water-source and water-sink composites of Dhole et al. (2006). Sorin and Bedard (2001) describe the evolutionary table method. The water surplus diagram was developed by Hallale (2002). The material recovery pinch diagram method was

independently developed by El-Halwagi et al. (2003) in the US and Prakash and Shenoy (2005) in India. Manan et al. (2004) developed the water cascade analysis (WCA) method and the method was improved upon by Foo et al. (2006). The algebraic targeting approach is an alternate pinch method which is also tabular in nature. It was developed by Almutlaq et al. (2005). Bandyopadhyay et al. (2006) introduced the source composite method and it is a technique which combines both algebraic and graphical approaches. The analytical method was developed by Liu et al. (2007). El-Halwagi and Hamad (1996) developed a mixed-integer linear programming method to generate waste-inception networks (WINs) which was improved upon by Doyle and Smith (1997). Alva-Argaez, Kokkosis and Smith (1998) extended the method of Doyle and Smith (1997). A fairly new automated technique has been developed by Ng et al. (2008) and is a mathematical optimisation technique based on the framework of the water cascade analysis technique.

Early methods for regeneration targeting are presented by Smith, Wang and Kuo (1994-1998), Wang and Smith (1998), Liu and Manan (1999), Deng et al. (2007-2008), Feng et al. (2007). Hallale (2002) first presented a guideline in placing regeneration units for fixed flow rate problems. Later works by El-Halwagi et al. (2006), Manan et al. (2004) and Foo et al. (2006) all follow this guideline presented by Hallale (2002). The ultimate flowrate targeting technique proposed by Ng et al. (2007, 2008) was the first method to determine the minimum fresh water, waste water and regeneration flowrate. A source composite curve method was developed by Bandyopadhyay and Cormos (2008). The ultimate flowrate targeting technique of Ng et al. (2008) was extended to regeneration targeting by Ng et al. (2009).

This research field for targeting for wastewater treatment has not been investigated in as much detail as the pinch-based approach. Bandyopadhyay and Cormos (2008) and Ng et al. (2007) have presented methods.

Kuo and Smith (1998) present the earliest method in targeting for the total water network. Ng et al. (2007) present a step-by-step procedure to determine the total water network for the fixed-rate type of problem.

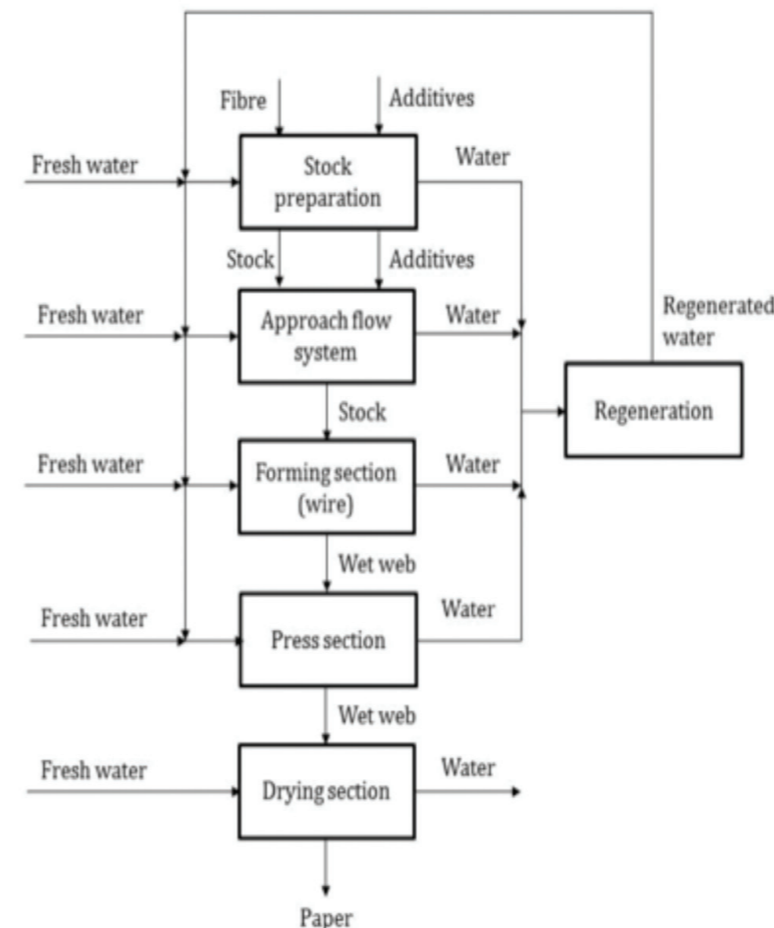
An automated targeting approach to determine the total water network has been developed by Linnhoff and March and has been trademarked WaterPinch and is available as a mathematical programming tool. Bedard et al. (2001) list various steps to be followed when approaching a water pinch problem.

2. The paper making process

Figure 1 is a basic flow diagram of the paper making process. It is observed that water is an integral part of the production process as it contributes to a major portion of the stock flow. It is used in large quantities and there is potential for more efficient water reuse, recycle and regeneration.

The paper making process is defined as fixed-rate type of problem because water is a part of the process rather than just being a medium for separating a certain fixed-load of contaminant from a stream of interest. In the fixed-rate type of problem a sink refers to when water of a particular quality is required to be added to a process and a source refers to water which is removed from a process which has the potential for reuse, recycle, regeneration and a combination thereof, therefore a water pinch approach was selected, to give an indication as to where regeneration would be most efficiently placed in a water using system.

Figure 1: A general flow diagram of the paper making process



3. Selecting an appropriate water pinch analysis method

The aim of the project (which commenced in 2010) was to find a suitable approach to systematically reduce the specific water consumption at a paper mill with associated reduction in the effluent produced from the paper making process. The significant factors under consideration were the total suspended solids concentration, equipment water quality constraints, plant space, economics and a long term solution.

Water-source and water-sink composites and the evolutionary table methods were not selected because they are iterative and these methods do not identify the limiting pinch point in a multiple pinch problem. The material recovery pinch diagram was not selected because it is difficult to identify the pinch point when the source-sink curves have very similar slopes. The algebraic method of Almutlaq et al. (2005) has not been as extensively applied as the other water pinch analysis methods. Where applied, no conclusive results have been obtained showing that the minimum water targets were determined and it was therefore not considered. The disadvantage of the source composite method of Bandyopadhyay and Cormos (2008) is that for systems where there is the discharge of wastewater from the system, the minimum regeneration flowrate cannot be determined. Ng et al. (2008) developed a mathematical optimisation based on the WCA. However it is relatively new and has not been extensively applied; it also cannot be applied to multiple fresh water sources or threshold problems. Considering all of this, the best suited method for the system was the ultimate flowrate targeting technique with regeneration placement of Ng et al. (2009).

4. The ultimate flowrate targeting technique with regeneration placement (Ng et al., 2009)

The ultimate flowrate targeting technique with regeneration placement of Ng et al. (2009) is a numerical method which is analogous to the heat problem table method developed for energy integration in process systems. Table 1 is the generic water cascade table presented by Ng et al. (2007), it can be used to target for a single pure or impure fresh-water feed. In Table 1:

- $F_{FW,k}$ = Interval feed water flowrate
- C_k = Concentration at level k
- C_{FW} = Fresh water concentration
- C = concentration
- k = concentration level
- n = last concentration level, usually the effluent concentration
- $C_{um,\Delta mk}$ = Cumulative impurity load

*Any set of consistent units can be used

Column								
1	2	3	4	5	6	7	8	9
k	C_k	$\sum_j F_j$	$\sum_i F_i$	$\sum_i F_i - \sum_j F_j$	$F_{C,k}$	Δm_k	Cum. Δm_k	$F_{FW,k}$
k	C_k	$(\sum_j F_j)_1$	$(\sum_i F_i)_1$	$(\sum_i F_i - \sum_j F_j)_1$	F_{FW}			
$k+1$	C_{k+1}	$(\sum_j F_j)_{k+1}$	$(\sum_i F_i)_{k+1}$	$(\sum_i F_i - \sum_j F_j)_{k+1}$	$F_{C,k}$	Δm_k		
					$F_{C,k+1}$	Δm_{k+1}	Cum. Δm_{k+1}	$F_{FW,k+1}$
\vdots	\vdots	\vdots	\vdots	\vdots	\vdots	\vdots	\vdots	\vdots
\vdots	\vdots	\vdots	\vdots	\vdots	\vdots	\vdots	\vdots	\vdots
\vdots	\vdots	\vdots	\vdots	\vdots	\vdots	\vdots	\vdots	\vdots
$n-2$	C_{n-2}	$(\sum_j F_j)_{n-2}$	$(\sum_i F_i)_{n-2}$	$(\sum_i F_i - \sum_j F_j)_{n-2}$	$F_{C,n-2}$	Δm_{n-2}		
$n-1$	C_{n-1}	$(\sum_j F_j)_{n-1}$	$(\sum_i F_i)_{n-1}$	$(\sum_i F_i - \sum_j F_j)_{n-1}$	$F_{C,n-1} = F_{WW}$	Δm_{n-1}	Cum. Δm_{n-1}	$F_{FW,n-1}$
n	C_n						Cum. Δm_n	$F_{FW,n}$

Table 1: Generic water cascade table of Ng et al. (2007)

- 4.1 In column 1(k) and 2 (C_k) the concentrations are arranged in ascending order.
- 4.2 j represents a sink. The flowrates of these sinks are summed at their respective concentration levels in column 3.
- 4.3 i represents a source. The flowrates of these sources are summed at their respective concentration levels in column 4.
- 4.4 In column 5, the nett flow rate between the sources and sinks are determined for each concentration level (k). A positive nett flow represents a surplus of water and a negative represents a deficit of water.
- 4.5 Next, the nett water flowrate surplus/deficit is cascaded down the concentration levels to yield the cumulative surplus/deficit flowrates (Column 6). Initially a fresh water feed (F_{FW}) is assumed to be zero. This is done to facilitate the search for the minimum water flowrates and this zero flow will be replaced once the rigorous fresh water target is located (Ng et al., 2007)
- 4.6 In column 7, the impurity load is determined. This is calculated from the product of the cumulative flowrate ($F_{C,k}$) and the concentration difference between two concentration levels i.e. $C_{k+1} - C_k$
- 4.7 These impurity loads calculated in column 7 are cascaded down in column 8 to produce the

- 4.8 cumulative impurity load. If a negative cumulative impurity load results in column 8, this means that an impurity load is transferred from a lower concentration to a higher concentration, which is not feasible. In this case, an interval feed water flowrate ($F_{FW,k}$) is determined in column 9. This is calculated from the following equation (Ng et al., 2007):

$$F_{FW,k} = \frac{Cum. \Delta m_k}{C_k - C_{FW}}$$

The negative flows which result indicate that due to the initial assumption of zero fresh water, there is not enough fresh water to meet all water demands. Hence, some fresh water is required to be added. To obtain the minimum fresh water flow required the absolute value of the largest negative $F_{FW,k}$ will then replace the earlier assumed zero fresh water flow in column 6 and a new set of feasible flowrate cascade and hence load cascade are determined. This new fresh water flowrate represents the minimum fresh water flowrate of the network. The final row in column 6 (F_{WW}) represents the wastewater flowrate generated by the network. (Ng et al., 2007)

There will exist at a certain concentration and purity level k a flow $F_{FW,k} = 0$. This concentration is referred to as the pinch.

The advantage of water cascade analysis and using the water cascade table is that it quickly yields the exact utility targets and the pinch location(s) without the iterative steps required by the water surplus diagram. The first advantage of WCA is that both F_{FW} and F_{WW} are identified. With the water surplus diagram only F_{FW} is determined from a long iterative procedure.

The second advantage is that it clearly identifies the pinch-causing stream and hence the exact water allocation for the regions above and below the pinch to achieve the minimum water targets during the network design (Foo et al., 2007).

Using the information obtained from the water cascade analysis, that is, the pinch point in the system, the most effective regeneration unit for the system can be identified. Using the regeneration potential of the units selected, the minimum regeneration flowrate can be targeted for by following the reallocation procedure described by Ng et al. (2009).

In the reallocation procedure, the water sources and sinks are separated into the fresh water region (FWR) and the regenerated water region (RWR). In the FWR the sinks will receive fresh water only and in the RWR the sinks will receive regenerated and process water. The water cascade analysis is re-performed in these two separate regions. The minimum fresh water flowrate is determined from the WCA of the FWR, the minimum regeneration flowrate is determined from the WCA on the RWR and the minimum effluent flowrate from the system is the combined effluent flowrate from the FWR and RWR.

5. Preliminary work required

In order for the results of the water cascade analysis to be accurate, it is necessary that an accurate representation of the mills paper making process is available and if not, then it is necessary to gather the important process. This will include process flow data, particularly the water flow data, the contaminant concentrations of the process water and the allowable contaminant concentration to a process unit.

Such contaminants include total suspended solids (TSS), total dissolved solids (TDS), biological oxygen demand (BOD), chemical oxygen demand (COD), etc.

For this research study, the contaminant of primary concern was the TSS with the cationic demand (CD) being the dependent contaminant.

6. Applying the water pinch analysis technique

Once the process data had been acquired the WCA was performed to determine the pinch causing stream on the paper machines. Depending on the pinch stream identified, an appropriate regeneration technology can be selected and then the water cascade analysis can be performed on the FWR and the RWR.

The pinch stream was identified as the accepts from the vibrating screens on one paper machine and water from the vacuum section on the other paper machine. The TSS of the vibrating screen accepts was significantly higher than the TSS of the water from the vacuum system and hence two different degrees of regeneration were required.

It was determined that a fine filtration system with a regeneration capability of producing a clarified water source of 20 ppm TSS would be suitable for the first paper machine whilst a microfiltration unit would be required for the second paper machine.

Assuming the placement of these units across the pinch point, the WCA was performed on the FWR and RWR on each of the paper machines to determine the ultimate water targets.

An example of application of the method to the fixed-rate type of problem can be viewed in the original works of Ng et al. (2007 – 2009).

7. Water network synthesis

Once the minimum water targets have been determined, a suitable water network must be developed in order to achieve the targets. Various water synthesis methods exist; the method selected for application in the research study was the Principle of Nearest Neighbours of Prakash and Shenoy (2005) and applying the nearest neighbour's algorithm. All equations used in the algorithm as well as the algorithm can be viewed in the original works of Prakash and Shenoy (2005).

8. Results obtained

Various water cascade analyses were performed on various reallocations of the source and sink streams into the FWR and RWR to determine if the method does indeed produce the minimum water targets as well as the global pinch point of a multiple pinch problem.

The results for each of the paper machines are available in Table 2 and Table 3 for papermachine one and papermachine two respectively.

Water pinch analysis results for paper machine one	
Initial water usage	21 m ³ .tonne ⁻¹
Minimum fresh water consumption as determined by WCA	7 m ³ .tonne ⁻¹
Pinch causing source	Vibrating screen accepts
Pinch concentration	1 885 ppm
Minimum effluent flowrate	5 000 l.min ⁻¹
Effluent concentration	1 000 000 ppm
Minimum regeneration flowrate	5 000 l.min ⁻¹
Regeneration concentration	20 ppm TSS
Regeneration unit	Fine-filtration unit
Total water reduction	14m ³ .tonne ⁻¹
Total fixed capital	R11.48 million
Total annual cost	R27.32 million
Total annual savings	R29.73 million
ROI	17%

Table 2: Water pinch analysis results for paper machine one

Water pinch analysis results for paper machine two	
Initial water usage	21 m ³ .tonne ⁻¹
Minimum fresh water consumption as determined by WCA	8 m ³ .tonne ⁻¹
Pinch causing source	Water from SBR chamber & main vacuum separator
Pinch concentration	6 ppm
Minimum effluent flowrate	900 l.min ⁻¹
Effluent concentration	1 000 000 ppm
Minimum regeneration flowrate	1 000 l.min ⁻¹
Regeneration concentration	0 ppm
Regeneration unit	Microfiltration unit
Total water reduction	13 m ³ .tonne ⁻¹
Total fixed capital	R3.07 million
Total annual cost	R2.18 million
Total annual savings	R3.27 million
ROI	30.39%

Table 3: Water pinch analysis results for paper machine two

9. Conclusions

- The problem faced at the paper plant, was that large quantities of water were being consumed by its paper machines, therefore the aim of the research study was to determine an effective approach to reduce the specific water consumption on these machines.
- A pinch analysis approach was selected to give some insight to the problem.
- An extensive literature survey was conducted and it was determined that the ultimate flowrate targeting technique with regeneration placement of Ng et al. (2009) was the most suitable method to apply to the mills paper making processes. This method was the most suitable at the commencement of the project, other works published during the project progression have not been considered.
- The contaminants of interest, limiting the total water reuse, were the total suspended solids and cationic demand.
- The ultimate flowrate targeting technique with regeneration placement was applied to both paper machine number one and paper machine

number two and the minimum fresh water, regenerated water and effluent flowrates were determined.

- The method also clearly identifies the global pinch point of a multiple pinch problem.
- The method of Ng et al. (2009) has identified a potential reduction of fresh water by 60 - 70 % on both the paper machines. This is with respect to the contaminants considered in the research study, if other parameters are considered the savings may reduce.
- It has been identified that a fine filtration system can replace all regeneration equipment on paper machine one if placed at the vibrating screen accepts.
- A microfiltration unit is required for paper machine number two if placed at the SBR and main vacuum separator.
- The water pinch analysis is very sensitive to the total suspended solids of the water from the vacuum system as well as the press and wire section.
- The nearest neighbour's algorithm of Prakash

and Shenoy (2005) was successful in developing a water network to achieve the minimum water targets determined by the method of Ng et al. (2009).

- Pinch analysis does prove to be an effective tool in determining the bottleneck in the system which limits the total water reuse in the system. Pinch analysis clearly identifies the problem streams such that these streams can be appropriately treated to reduce the specific water consumption. It provides a transparent solution.
- As mentioned, the contaminants of interest were the total suspended solids and the cationic demand. As the water network is closed there could be possible accumulation of contaminants which were not considered in this research study. It was observed that there will be an increase in cationic demand if left untreated. This could also occur with BOD and COD. There could be an added adverse effect of scaling at the drying section due to the increased recycle of process water and cycling up of contaminants such as calcium ions.
- Due to the increased recycling of process water, there could be an increase in the temperature profile across both the machines, which has a benefit in retention and lower energy consumption.
- Water pinch analysis can be successfully applied to any type of process industry provided that the process is correctly identified as a constant rate or constant load type of problem and then the appropriate respective pinch analysis method selected and applied. The added requirement is that a verified process balance with the contaminants of interest must be available.
- The advantage of the process integration approach is that it provides system transparency and identifies the most efficient placing of regeneration units and connection between units to achieve reduced water consumption. This has major implications on all processes which use water in larger quantities, not only the pulp and paper industry.

10. Recommendations

This research study into reducing the specific water consumption at the paper machines at a South African

paper plant was based on total suspended solids being the contaminant of interest with respect to total water reuse in the system; all results obtained are therefore in respect to the total contaminant concentration. In light of this the following is recommended:

- The vacuum system, wire and press section must be carefully monitored to ensure that the systems are operating at the optimum operating conditions
- Investigations should be performed on a laboratory scale to determine how other properties, such as BOD and COD, will accumulate in the system as the water network is closed. Also, to determine the effect on scaling or corrosion, slime growth, pitch and deposits due to increased process water recirculation.
- This study has shown that water pinch analysis can be an effective tool to reduce the specific water consumption therefore in order to incorporate more detailed operating parameters and system properties into the analysis, an investment should be made in a pulp and paper simulation package (many are available) so that various properties can be determined from the simulation and then used in the pinch analysis. Various water and energy pinch analysis packages are also available. ■

Abbreviations

- BOD : Biological oxygen demand
- COD : Chemical oxygen demand
- FWR : Fresh Water Region. The sources and sinks in the FWR are those which have a concentration lower than the concentration of the water stream leaving the regeneration unit
- RWR : Regenerated Water Region: The sources and sinks in the RWR are those which have concentrations higher than that of the regeneration unit
- WCA : Water Cascade Analysis: Numerical procedure used to determine the minimum water targets
- WCT : Water cascade table: Used in the water cascade analysis technique

Subscripts:

- i* source
- j* sink
- k* level
- m* Number of demands (sinks)
- n* Number of sources

Nomenclature

$F_{c,k}$	Cumulative surplus/deficit flowrates (L.min ⁻¹)
$F_{FW,k}$	Interval feed water flowrate (L.min ⁻¹)
$F_{i,A}$	Additional source flow to be reallocated to the FWR, which has to be equal to the additional sink flow since these must be equal to avoid imbalance in the RWR (L.min ⁻¹)
Δm_k	Impurity load (L.min ⁻¹)
C	Concentration (mg.L ⁻¹)
$C_{i,A}$	Limiting concentration of the additional source to be allocated to the FWR (mg.L ⁻¹)
$C_{j,A}$	Limiting concentration of the additional sink to be allocated to the FWR (mg.L ⁻¹)
C_{pinch}	Pinch concentration (ppm)
$Cum\Delta m_k$	Cumulative impurity load (L.min ⁻¹)
D	Demand
D_p	Demand of given concentration level
F	Flowrates (L.min ⁻¹)
F_{FW}	Minimum fresh water flowrate (L.min ⁻¹)
$F_j C_j$	Additional load which can be accepted by the sink in the FWR (L.min ⁻¹)
F_{WW}	Minimum wastewater flowrate (L.min ⁻¹)
S	Source
$S(k+1)$	Source with contaminant concentration just higher than that of D_p
S_k	Source with contaminant concentration just lower than the concentration of demand D_p

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